Chapter 12: Introduction to Analysis of Variance

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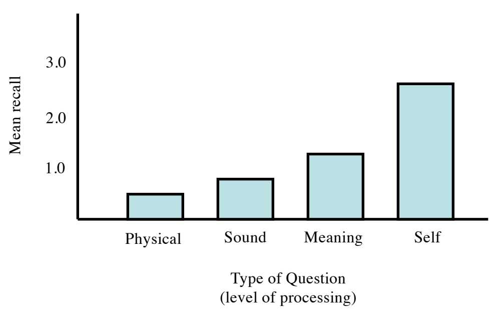

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(a)

Independent Variable: Age		
4 Years	5 Years	6 Years
Vocabulary scores for sample 1	Vocabulary scores for sample 2	Vocabulary scores for sample 3

(b)

		Independent Variable 1: Class Size		
		Small Class	Medium Class	Large Class
Independent Variable 2: Teaching Method	Method A	Sample 1	Sample 2	Sample 3
	Method B	Sample 4	Sample 5	Sample6

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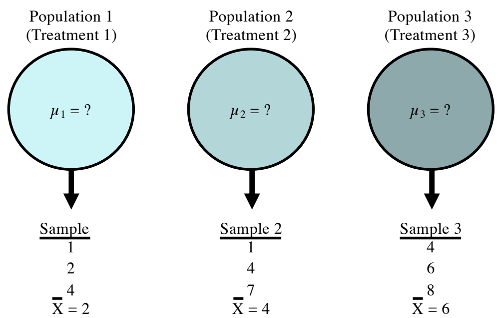

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Statistical Hypothesis (Null) for ANOVA

H_O : $\mu_1 = \mu_2 = \mu_3$ (There is no effect of…)

H_1 : At least one population mean is different from the others

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$$t = \frac{\text{Obtained difference between sample means}}{\text{Difference expected by chance (error)}}$$

$$F = \frac{\text{Variance (average squared differences) between sample means}}{\text{Variance (differences) expected by chance (sampling error)}}$$

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Treatment 1 50º (Sample 1)	Treatment 2 70º (Sample 2)	Treatment 3 90º (Sample 3)
0	4	1
1	3	2
3	6	2
1	3	0
0	4	0
$\overline{X} = 1$	$\overline{X} = 4$	$\overline{X} = 1$

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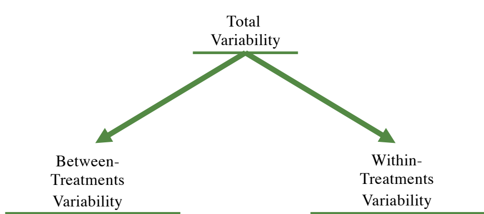

1. Treatment Effect
2. Individual Differences
3. Experimental Error

1. Individual Differences
2. Experimental Error

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$$F = \frac{\text{Variance between treatments}}{\text{Variance within treatments}}$$

$$= \frac{\text{treatment effect + individual differences + error}}{\text{individual differences + error}}$$

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Temperature Conditions			
1	2	3	
50º	70º	90º	
0	4	1	$\Sigma X^2 = 106$
1	3	2	G = 30
3	6	2	N = 15
1	3	0	k = 3
0	4	0	
$T_1 = 5$	$T_2 = 20$	$T_3 = 5$	
$SS_1 = 6$	$SS_2 = 6$	$SS_3 = 4$	
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$	
$\overline{X}_1 = 1$	$\overline{X}_2 = 4$	$\overline{X}_3 = 1$	

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SS total

SS between   SS within

df total

df between   df within

$$\text{Variance Between Treatments} = \frac{\text{SS between}}{\text{df between}}$$

$$\text{Variance Within Treatments} = \frac{\text{SS within}}{\text{df within}}$$

$$F = \frac{\text{Variance between treatments}}{\text{Variance within treatments}}$$

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SS Total

$$\Sigma X^2 - \frac{G^2}{N}$$

SS Between Treatments

$$\sum\frac{T^2}{n} - \frac{G^2}{N}$$

SS Within Treatments

ΣSS inside each treatment

12

df Total
$N-1$

df Between Treatments
$k-1$

df Within Treatments
$N-k$

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Total
$$SS = \Sigma X^2 - \frac{G^2}{N}$$
$$df = N-1$$

Between Treatments
$$SS = \sum \frac{T^2}{n} - \frac{G^2}{N}$$
$$df = k-1$$
$$MS = \frac{SS}{df}$$

Within Treatments
$$SS = \sum SS_{each\ \ treatment}$$
$$df = N-k$$
$$MS = \frac{SS}{df}$$

$$F-ratio = \frac{MS_{between\ \ treatments}}{MS_{within\ \ treatments}}$$

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Source	SS	df	MS	F
Between Treatments	30	2	15	F(2,12) = 11.28
Within Treatments	16	12	1.33	
Total	46	14		

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Source	SS	df	MS	F	p < .05
Between Treatments (Temp.)	30	2	15	F(2,12) = 11.28	✓
Within Treatments	16	12	1.33		
Total	46	14			

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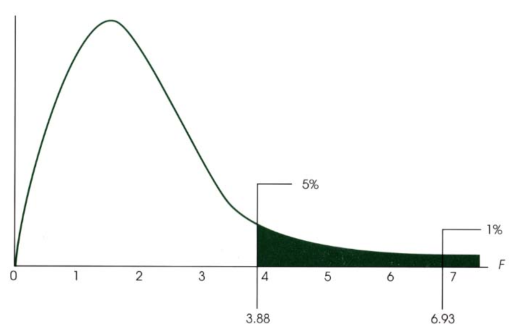

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Degrees of Freedom Denominator	Degrees of Freedom : Numerator					
	1	2	3	4	5	6
10	4.96 10.04	4.10 7.56	3.71 6.55	3.48 5.99	3.33 5.64	3.22 5.39
11	4.84 9.65	3.98 7.20	3.59 6.22	3.36 5.67	3.20 5.32	3.09 5.07
12	4.75 9.33	3.88 6.93	3.49 5.95	3.26 5.41	3.11 5.06	3.00 4.82
13	4.67 9.07	3.80 6.70	3.41 5.74	3.18 5.20	3.02 4.86	2.92 4.62
14	4.60 8.86	3.74 6.51	3.34 5.56	3.11 5.03	4.96 4.69	2.85 4.46

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Degrees of Freedom Denominator	Degrees of Freedom : Numerator					
	1	2	3	4	5	6
10	4.96 10.04	4.10 7.56	3.71 6.55	3.48 5.99	3.33 5.64	3.22 5.39
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Pain Tolerance Study Data

Placebo	Drug A	Drug B	Drug C	
0	0	3	8	N = 12
0	1	4	5	G = 36
3	2	5	5	$\Sigma x^2 = 178$
T = 3	T = 3	T = 12	T = 18	
SS = 6	SS = 2	SS = 2	SS = 6	

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Pain Tolerance Study Data

Placebo	Drug A	Drug B	Drug C		
0	0	3	8	N = 12	$G^2/N = 108$
0	1	4	5	G = 36	
3	2	5	5	$\Sigma x^2 = 178$	
T = 3	T = 3	T = 12	T = 18		
SS = 6	SS = 2	SS = 2	SS = 6		
$n_1 = 3$	$n_2 = 3$	$n_3 = 3$	$n_4 = 3$		
$\bar{x}_1 = 1$	$\bar{x}_2 = 1$	$\bar{x}_3 = 4$	$\bar{x}_4 = 6$		

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Placebo: $SS_1 = \Sigma(x - \overline{X})^2$

$$SS_1 = (0-1)^2 + (0-1)^2 + (3-1)^2$$
$$= 1 + 1 + 4$$
$$= 6$$

or $SS_1 = \sum x^2 - \frac{(\sum x)^2}{n}$

$$SS_1 = (0+0+9) - \frac{(3)^2}{3}$$
$$= 9 - 3$$
$$= 6$$

Drug A : $SS_2 = \sum x^2 - \frac{(\sum x)^2}{n}$

$$SS_2 = (0^2 + 1^2 + 2^2) - \frac{(3)^2}{3}$$
$$= 5 - 3$$
$$= 2$$

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Drug B : $SS_3 = \sum x^2 - \frac{(\sum x)^2}{n}$

$$SS_3 = (3^2 + 4^2 + 5^2) - \frac{(12)^2}{3}$$
$$= (9 + 16 + 25) - 48$$
$$= 50 - 48$$
$$= 2$$

Drug C : $SS_4 = \sum x^2 - \frac{(\sum x)^2}{n}$

$$SS_4 = (8^2 + 5^2 + 5^2) - \frac{(18)^2}{3}$$
$$= (64 + 25 + 25) - 108$$
$$= 114 - 108$$
$$= 6$$

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$$F = \frac{MS_{between}}{MS_{within}} = \frac{\frac{54}{3}}{\frac{16}{8}} = \frac{18}{2} = 9.00$$

$F(3,8) = 9.00,\ p < .05$

Source	SS	df	MS	F	p < .05
Between Treatments	54	3	18	F(3,8) = 9.00	✓
Within Treatments	16	8	2		
Total	70	11			

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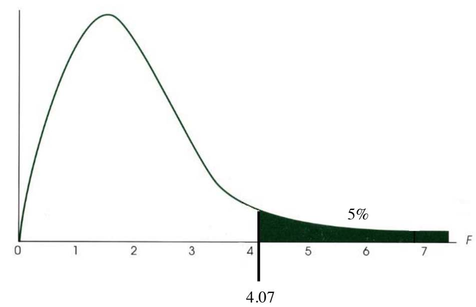

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Reporting the results for the Pain Tolerance Study

The average length of time participants were able to tolerate a painful stimulus for each of the different drug conditions are presented in Table 1. A single-factor analysis of variance confirmed an overall effect of drug type on pain tolerance, $F(3,8) = 9.00$, $MSE = 2.00$, $p < .05$.

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Table 1. Average time (seconds) a painful stimulus was endured for different drug treatment conditions.

	Treatment Condition			
	Placebo	Drug A	Drug B	Drug C
M	1.0	1.0	4.0	6.0
SD	1.73	1.00	1.00	1.73

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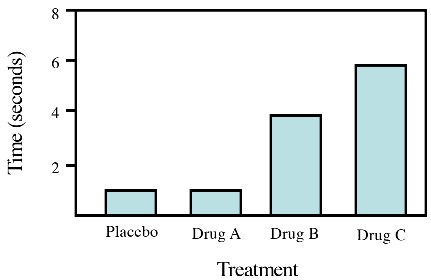
Average tolerance of a painful stimulus as a function of drug treatment condition



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Post Hoc Tests

After ANOVA when:

1. You reject H_o and…
2. There are 3 or more treatments ($k \geq 3$)

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Tukey's Honestly Significant Difference Test (or HSD)

Denominator of F-ratio

$$HSD = q\sqrt{\frac{MS_{within}}{n}}$$

From Table (# of treatments, df_{within})

Number of Scores in Each Treatment

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Scheffe Test

1. Conservative - safest of all post hoc tests
2. Compute a new F-ratio for differences between any pair of means
3. $$F = \frac{MS_{between} \text{ (just for the pair of means tested)}}{MS_{within} \text{ (from the overall ANOVA)}}$$

a) Use k from overall to compute $df_{between}$, therefore $df_{between} = k - 1$

b) Critical F same as for the overall test

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Placebo	Drug A	Drug B	Drug C
n = 3	n = 3	n = 3	n = 3
T = 3	T = 3	T = 12	T = 18
$\overline{X} = 1$	$\overline{X} = 1$	$\overline{X} = 4$	$\overline{X} = 6$

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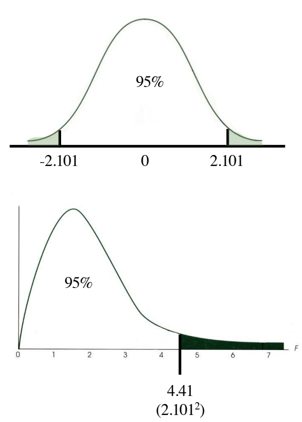

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Assumptions for Independent Measures ANOVA

1. Observations in each sample are independent.
2. Populations from which samples are selected must be normal.
3. Populations from which samples selected must have equal variances (homogeneity of variance)

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Testing Homogeneity of Variance: Hartley's F-max test

1. For independent measures designs
2. Compute sample variances for each sample:

$$s^2 = \frac{SS}{df}$$

3. $F_{\max} = \dfrac{s^2(largest)}{s^2(smallest)}$

4. Compare the F-max obtained with the critical value in Table B3
   a) k = number of samples
   b) df = n-1 for each sample variance (equal sample sizes)
   c) α level

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The performance of different species of monkeys on a delayed response task.

Vervet	Rhesus	Baboon	
n = 4	n = 10	n = 6	N = 20
$\overline{X} = 9$	$\overline{X} = 14$	$\overline{X} = 4$	G = 200
T = 36	T = 140	T = 24	$\Sigma x^2 = 3400$
SS = 200	SS = 500	SS = 320	

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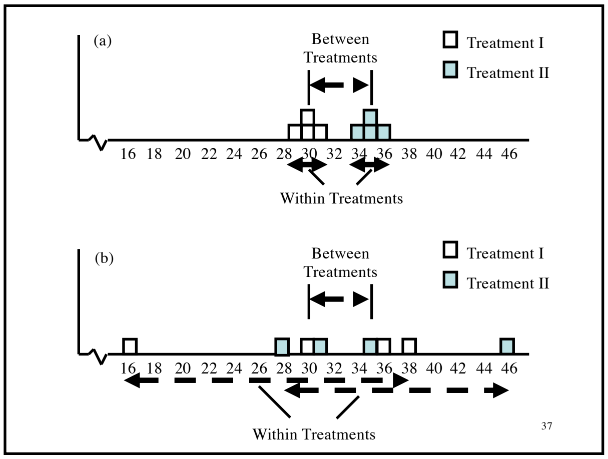

(a)
Between
Treatments
Treatment I
Treatment II
16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46
Within Treatments
(b)
Between
Treatments
Treatment I
Treatment II
16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46
Within Treatments
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