# Chapter 13: Repeated-Measures Analysis of Variance

## The Logical Background for a Repeated-Measures ANOVA

- Chapter 14 extends analysis of variance to research situations using repeated-measures (or related-samples) research designs.
- Much of the logic and many of the formulas for repeated-measures ANOVA are identical to the independent-measures analysis introduced in Chapter 13.
- However, the repeated-measures ANOVA includes a second stage of analysis in which variability due to **individual differences** is subtracted out of the error term.

## The Logical Background for a Repeated-Measures ANOVA (cont.)

- The repeated-measures design eliminates individual differences from the between-treatments variability because the same subjects are used in every treatment condition.
- To balance the F-ratio the calculations require that individual differences also be eliminated from the denominator of the F-ratio.
- The result is a test statistic similar to the independent-measures F-ratio but with all individual differences removed.

IM ANOVA:

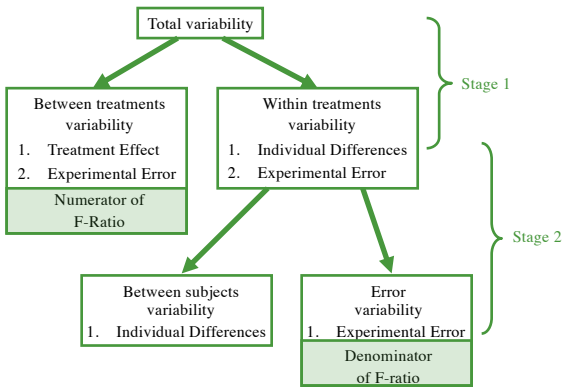
$$F = \frac{\text{Variance Between Treatments}}{\text{Variance Due to Chance}}$$

$$= \frac{\text{Variance (differences) Between Treatments}}{\text{Variance (differences) Expected Sampling Error}}$$

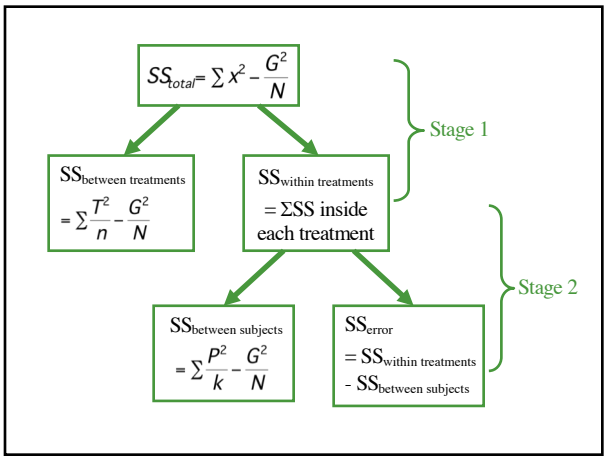
$$= \frac{\text{treatment effect + individual differences + error}}{\text{individual differences + error}}$$

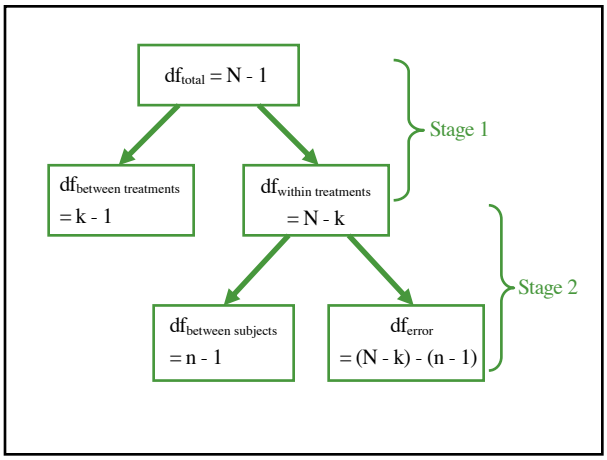
RM ANOVA:

$$F = \frac{\text{Treatment Effect + Experimental Error}}{\text{Experimental Error}}$$

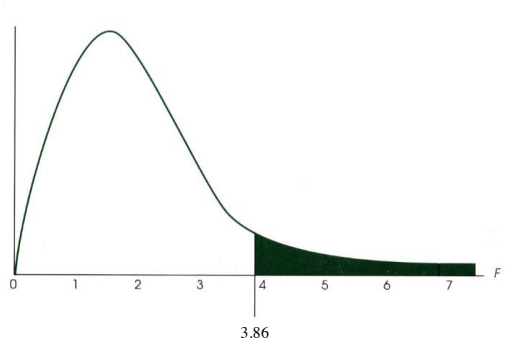



| | Test Session | | | |
|---|---|---|---|---|
| Person | Session 1 | Session 2 | Session 3 | p |
| A | 3 | 3 | 6 | 12 |
| B | 2 | 2 | 2 | 6 |
| C | 1 | 1 | 4 | 6 |
| D | 2 | 4 | 6 | 12 |
| | $T_1 = 8$ | $T_2 = 10$ | $T_3 = 18$ | |
| | $SS_1 = 2$ | $SS_2 = 5$ | $SS_3 = 11$ | |
| $G = 36$ | $\Sigma x^2 = 140$ | $k = 3$ | $n = 4$ | $N = 12$ |




| Subject | Before Treatment | One Week Later | One Month Later | Six Months later | p |
|---|---|---|---|---|---|
| A | 8 | 2 | 1 | 1 | 12 |
| B | 4 | 1 | 1 | 0 | 6 |
| C | 6 | 2 | 0 | 2 | 10 |
| D | 8 | 3 | 4 | 1 | 16 |
| | $T_1 = 26$ | $T_2 = 8$ | $T_3 = 6$ | $T_4 = 4$ | |
| | $SS_1 = 11$ | $SS_2 = 2$ | $SS_3 = 9$ | $SS_4 = 2$ | |
| | n = 4 k = 4 | N = 16 | G = 44 | $\Sigma x^2 = 222$ | |

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| A | 8 | 2 | 1 | 1 | 12 |
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| D | 8 | 3 | 4 | 1 | 16 |
| | $T_1 = 26$ | $T_2 = 8$ | $T_3 = 6$ | $T_4 = 4$ | |
| | $SS_1 = 11$ | $SS_2 = 2$ | $SS_3 = 9$ | $SS_4 = 2$ | |
| | $n = 4$ $k = 4$ | $N = 16$ | $G = 44$ | $\Sigma x^2 = 222$ | |
| | $\bar{X}_1 = 6.5$ | $\bar{X}_2 = 2.0$ | $\bar{X}_3 = 1.5$ | $\bar{X}_4 = 1.0$ | |

## Tukey's Honestly Significant Difference Test (or HSD) for Repeated Measures ANOVA

Denominator of F-ratio

$$HSD = q\sqrt{\frac{MS_{error}}{n}}$$

From Table (Number of treatments, $df_{error}$)

Number of Scores in each Treatment

## Advantages of Repeated Measures Design

1. Economical - fewer SS required
2. More sensitive to treatment effect - individual differences having been removed

Independent:

$$F = \frac{\text{treatment effect + individual differences + experimental error}}{\text{individual differences + experimental error}}$$

vs.

Repeated Measures:

$$F = \frac{\text{treatment effect + experimental error}}{\text{experimental error}}$$

Imagine: Treatment Effect = 10 units of variance
Individual Differences = 1000 units of variance
Experimental Error = 1 unit of variance

## Disadvantages of Repeated Measures Designs:

1. Carry over effects (e.g. drug 1 vs. drug 2)
2. Progressive error (e.g. fatigue, general learning strategies, etc.)

*Counterbalancing

## Assumptions of the Repeated Measures ANOVA

1. Observations within each treatment condition must be independent
2. Population distribution within each treatment must be normal
3. Variances of the population distributions for each treatment must be equivalent (homogeneity of variance)
4. Homogeneity of covariance.