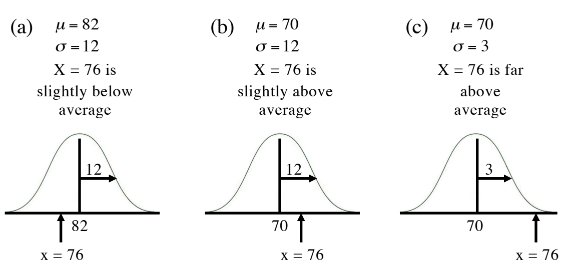
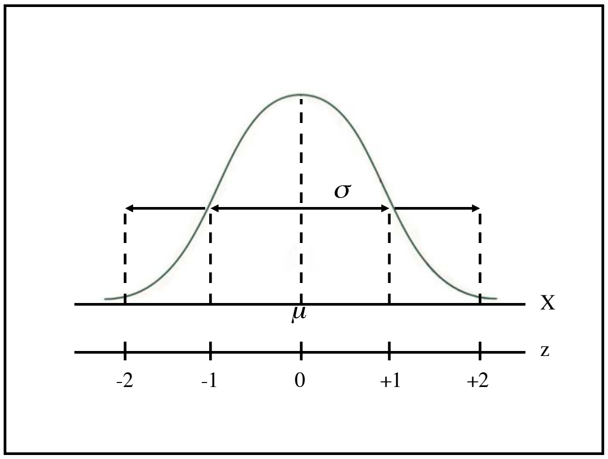
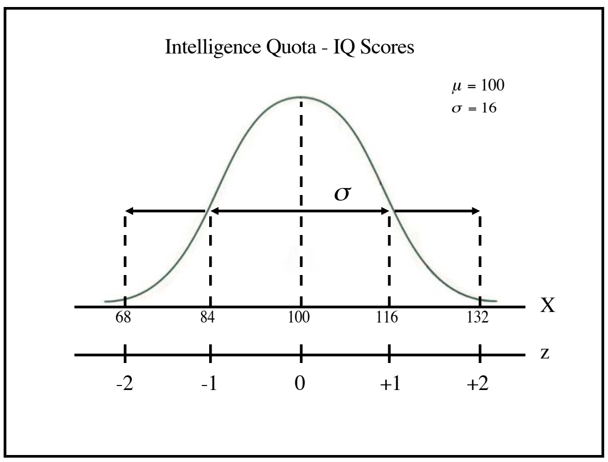
Chapter 5: z-Scores

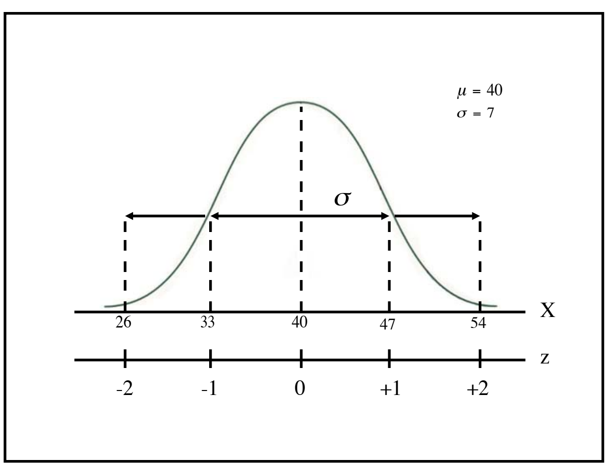


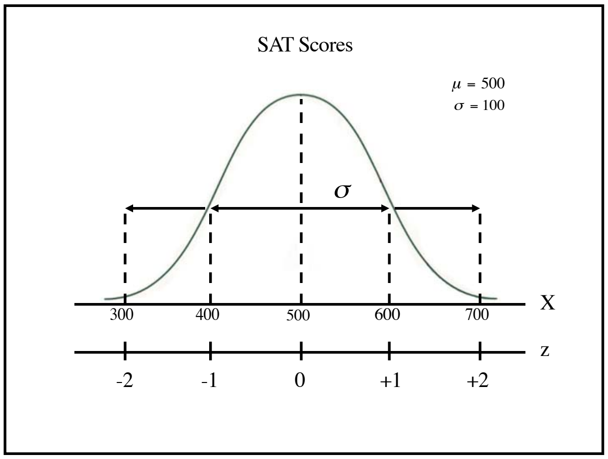
Definition of z-score

- A z-score specifies the precise location of each x-value within a distribution. The sign of the z-score (+ or -) signifies whether the score is above the mean (positive) or below the mean (negative). The numerical value of the z-score specifies the distance from the mean by counting the number of standard deviations between X and μ.

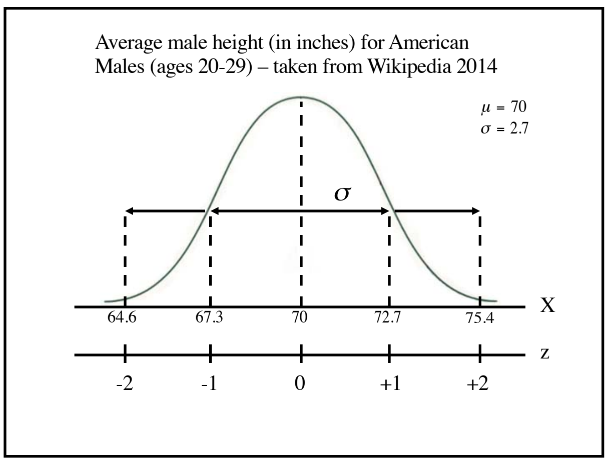

σ
X
μ
z
-2
-1
0
+1
+2


Intelligence Quota - IQ Scores
μ = 100
σ = 16
σ
X
68
84
100
116
132
z
-2
-1
0
+1
+2

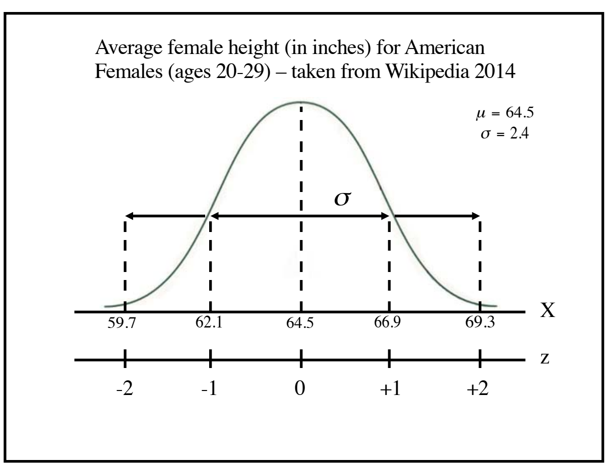

μ = 40
σ = 7
σ
X
26
33
40
47
54
z
-2
-1
0
+1
+2



SAT Scores
μ = 500
σ = 100
σ
X
300 400 500 600 700
z
-2 -1 0 +1 +2



Average male height (in inches) for American Males (ages 20-29) – taken from Wikipedia 2014
μ = 70
σ = 2.7
σ
X
64.6 67.3 70 72.7 75.4
z
-2 -1 0 +1 +2

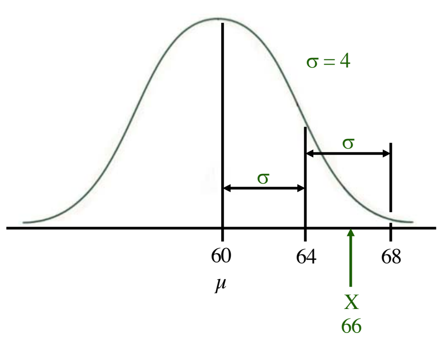


Average female height (in inches) for American Females (ages 20-29) – taken from Wikipedia 2014
μ = 64.5
σ = 2.4
σ
X
59.7 62.1 64.5 66.9 69.3
z
-2 -1 0 +1 +2

Example 5.2

- A distribution of exam scores has a mean (μ) of 50 and a standard deviation (σ) of 8.

$$z = \frac{x - \mu}{\sigma}$$



Example 5.5

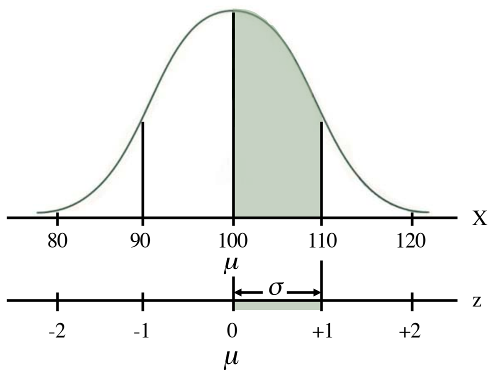
- A distribution has a mean of μ = 40 and a standard deviation of σ = 6.

To get the raw score from the z-score:

$$x = \mu + z\sigma$$

If we transform <u>every</u> score in a distribution by assigning a z-score, <u>new</u> distribution:

1. Same shape as original distribution
2. Mean for the new distribution will be <u>zero</u>
3. The standard deviation will be equal to 1



A small population

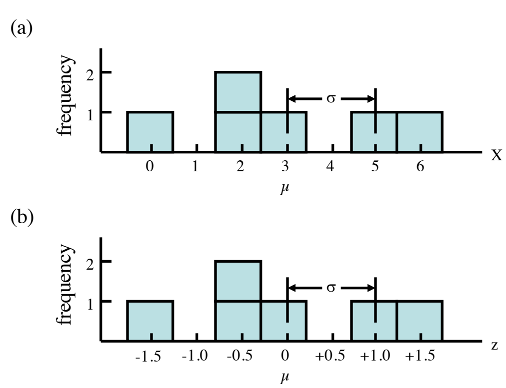
0, 6, 5, 2, 3, 2 N=6

x	x - μ	(x - μ)^2
0	0 - 3 = -3	9
6	6 - 3 = +3	9
5	5 - 3 = +2	4
2	2 - 3 = -1	
3	3 - 3 = 0	0
2	2 - 3 = -1	1

$\sum(x-\mu)^2 = SS = 24$

$$\mu = \frac{\sum x}{N} = \frac{18}{6} = 3$$

$$\sigma = \sqrt{\frac{SS}{N}} = \sqrt{\frac{\sum(x-\mu)^2}{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$



Let's transform every raw score into a z-score using: $z = \frac{x-\mu}{\sigma}$

x = 0	$z = \frac{0-3}{2}$	= -1.5
x = 6	$z = \frac{6-3}{2}$	= +1.5
x = 5	$z = \frac{5-3}{2}$	= +1.0
x = 2	$z = \frac{2-3}{2}$	= -0.5
x = 3	$z = \frac{3-3}{2}$	= 0
x = 2	$z = \frac{2-3}{2}$	= -0.5

Mean of z-score distribution :

$$\mu_z = \frac{\sum z}{N} = \frac{(-1.5)+(1.5)+(1.0)+(-0.5)+(0)+(-0.5)}{6} = 0$$

Standard deviation: $\sigma_z = \sqrt{\frac{SS_z}{N}} = \sqrt{\frac{\sum(x-\mu_z)^2}{N}}$

z	z - μ_z	(z - μ_z)^2
-1.5	-1.5 - 0 = -1.5	2.25
+1.5	+1.5 - 0 = +1.5	2.25
+1.0	+1.0 - 0 = +1.0	1.00
-0.5	-0.5 - 0 = -0.5	0.25
0	0 - 0 = 0	0
-0.5	-0.5 - 0 = -0.5	0.25
		$6.00 = \sum(z-\mu_z)^2$

$$\sigma = \sqrt{\frac{SS}{N}} = \sqrt{\frac{6}{6}} = \sqrt{1} = 1$$

What do we use z-scores for?

- Can compare performance on two different scales (e.g. compare your score on the ACT to your score on SAT) by converting scores to z scores and comparing z scores
- Can convert a distribution of scores with a specific mean and standard deviation to completely new distribution with a new mean and standard deviation

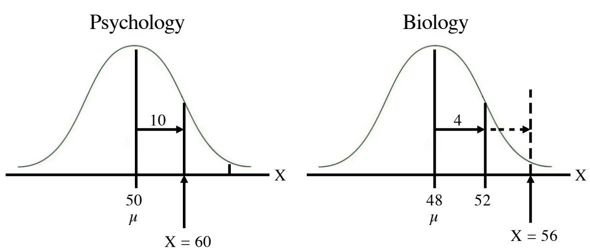
Comparing test scores on two different scales

Meghan’s Semester Test Results

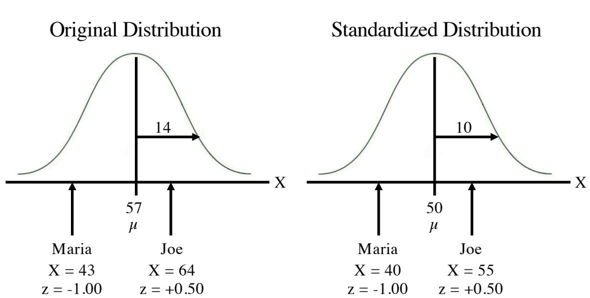
- Psychology Exam: score of 60
- Biology Exam: score of 56

- Which was her better score, relative to the others in each class?

To answer the question convert her scores to standard scores and compare—explain your answer fully in terms of z scores and standard deviations



Converting Distributions of Scores



Correlating Two Distributions of Scores with z-scores